# The multicomponent q-coherent states of the quantum algebra $sl_q(3)$



1994 J. Phys. A: Math. Gen. 27 3073

(http://iopscience.iop.org/0305-4470/27/9/021)

J. Phys. A: Math. Gen. 27 (1994) 3073–3081. Printed in the UK

# The multicomponent $q$-coherent states of the quantum algebra $sl_q(3)$

Guang-Hua Li†‡, San-Ru Hao‡, Jun-Yun Long‡ and Qian-Jun Yue§

† CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China
‡ Department of Physics, Changsha Normal University of Water Resources and Electric Power, Changsha, Hunan 410077, People's Republic of China
§ Computer Centre, Changsha Normal University of Water Resources and Electric Power, Changsha, Hunan 410077, People's Republic of China

Received 23 August 1993, in final form 23 December 1993

**Abstract.** The multicomponent $q$-coherent states associated with the quantum algebra $sl_q(3)$ are presented. The expansions are discussed for arbitrary operators in terms of the multicomponent $q$-coherent states of $sl_q(3)$. An inhomogeneous differential realization of $sl_q(3)$ in this multicomponent $q$-coherent state space is obtained.

## 1. Introduction

It is well known that the theory of coherent states has as long a history as quantum mechanics itself, and continues to generate interest, not just theoretically but also practically [1]. The concept of coherent states was first introduced by Schrödinger in 1926 [2]. The coherent states were first used by Glauber in the field of quantum optics [3], and extended to arbitrary Lie groups by Perelomov and Gilmore [4, 5]. An excellent review is given in [6].

Recently, the coherent states of quantum algebra have also attracted much attention. The $q$-deformed boson realization of the quantum algebra $su_q(2)$ has been introduced by Biedenharn [7] and Macfarlane [8]. The $q$-coherent states of the $q$-harmonic oscillator have been given by Biedenharn [7]. For the quantum algebra $su_q(2)$, the $q$-analogue of the usual spin coherent state has been constructed by Quesne [9].

In this paper, by analysing the properties of the finite-dimensional irreducible representations of $sl_q(3)$, we present the multicomponent $q$-coherent states of $sl_q(3)$, and discuss the expansions of arbitrary operators in terms of these $q$-coherent state vectors. We also study the inhomogeneous differential realization of $sl_q(3)$ in this multicomponent $q$-coherent state space.

## 2. Quantum algebra $sl_q(3)$

For the quantum algebra $sl_q(3)$, the general relations have been given by Jimbo [10]. The generators of $sl_q(3)$ have been rewritten as $Q$, $J_0$, $J_\pm$, $T_{\pm1/2}$ and $V_{\pm1/2}$ by Yu [11].

  

They obey the commutator relations

$$[Q, J_0]=[Q, J_\pm]=0 \qquad [J_0, J_\pm]=\pm J_\pm \qquad [J_+, J_-]=[2J_0]$$
$$[J_0, T_s]=sT_s \qquad [J_0, V_s]=sV_s \qquad [Q, T_s]=3T_s \tag{1}$$
$$[Q, V_s]=-3V_s \qquad s=\pm\tfrac{1}{2}$$

and

$$J_+=(J_-)^+ \qquad V_{\pm 1/2}=\mp(T_{\mp 1/2})^+ \qquad J_\pm^2 T_{\mp 1/2}+T_{\mp 1/2}J_\pm^2=[2]J_\pm T_{\mp 1/2}J_\pm \tag{2}$$

where

$$[x]=(q^x-q^{-x})/(q-q^{-1}) \tag{3}$$

so

$$[-x]=-[x]. \tag{4}$$

This tells us that the operators $J_0$ and $J_\pm$ form the quantum subalgebra $su_q(2)$ of the quantum algebra $sl_q(3)$. The operator $Q$ is an irreducible tensor of rank zero, $\{J_0, J_\pm\}$ is a set of irreducible tensor operators of rank 1, and $\{T_{\pm 1/2}\}$ and $\{V_{\pm 1/2}\}$ are two sets of rank $\frac{1}{2}$ for $su_q(2)$.

If $q$ is not a root of unity, the finite-dimensional irrep of $sl_q(3)$ can be given by the non-negative integers $\lambda$ and $\mu$. The bases of Hilbert space $V^{(\lambda\mu)}$ carrying the $IR(\lambda\mu)$ of $sl_q(3)$ are the Elliott-like bases $|(\lambda\mu)\varepsilon JM\rangle$, where $\varepsilon=-\lambda-2\mu,\ -\lambda-2\mu+3,\ldots,$ $2\lambda+\mu$; $J=\frac{1}{6}|2\lambda-2\mu-\varepsilon|,\ \frac{1}{6}|2\lambda-2\mu-\varepsilon|+1,\ldots,\ \min\{\frac{1}{6}(2\lambda+4\mu-\varepsilon),\ \frac{1}{6}(2\mu+4\lambda+\varepsilon)\}$ and $M=-J, -J+1,\ldots, J$ [12]. They are orthonormal and complete

$$\langle(\lambda\mu)\varepsilon JM|(\lambda'\mu')\varepsilon'J'M'\rangle=\delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{\varepsilon\varepsilon'}\delta_{JJ'}\delta_{MM'} \tag{5}$$

$$\sum_{\varepsilon JM}|(\lambda\mu)\varepsilon JM\rangle\langle(\lambda\mu)\varepsilon JM|=I \tag{6}$$

where $I$ is the identity operator. From [11], it can be shown without difficulty that

$$Q|(\lambda\mu)\varepsilon JM\rangle=\varepsilon|(\lambda\mu)\varepsilon JM\rangle$$
$$J_0|(\lambda\mu)\varepsilon JM\rangle=M|(\lambda\mu)\varepsilon JM\rangle$$
$$J_\pm|(\lambda\mu)\varepsilon JM\rangle=\{[J\mp M][J\pm M+1]\}^{1/2}|(\lambda\mu)\varepsilon JM\pm 1\rangle$$
$$T_{\pm 1/2}|(\lambda\mu)\varepsilon JM\rangle$$
$$=A(\varepsilon J)\{[J\pm M+1]\}^{1/2}|(\lambda\mu)\varepsilon+3J+\tfrac{1}{2}M\pm\tfrac{1}{2}\rangle$$
$$\mp B(\varepsilon J)\{[J\mp M]\}^{1/2}|(\lambda\mu)\varepsilon+3J-\tfrac{1}{2}M\pm\tfrac{1}{2}\rangle \tag{7}$$
$$V_{\pm 1/2}|(\lambda\mu)\varepsilon JM\rangle$$
$$=C(\varepsilon J)\{[J\pm M+1]\}^{1/2}|(\lambda\mu)\varepsilon-3J+\tfrac{1}{2}N\pm\tfrac{1}{2}\rangle$$
$$\pm D(\varepsilon J)\{[J\mp M]\}^{1/2}|(\lambda\mu)\varepsilon-3J-\tfrac{1}{2}M\pm\tfrac{1}{2}\rangle$$

where

$$A(\varepsilon J)=\{[\tfrac{1}{6}(2\lambda+4\mu-\varepsilon)-J][\tfrac{1}{6}(2\mu-2\lambda+\varepsilon)+J+1]$$
$$\times[\tfrac{1}{6}(4\lambda+2\mu+\varepsilon)+J+2][2J+1]^{-1}[2J+2]^{-1}\}^{1/2}$$

$$B(\varepsilon J)=\{[\tfrac{1}{6}(2\lambda-2\mu-\varepsilon)+J][\tfrac{1}{6}(2\lambda+4\mu-\varepsilon)+J+1]$$
$$\times[\tfrac{1}{6}(4\lambda+2\mu+\varepsilon)-J+1][2J]^{-1}[2J+1]^{-1}\}^{1/2} \tag{8}$$
$$C(\varepsilon J)=\{[\tfrac{1}{6}(2\lambda-2\mu-\varepsilon)+J+1][\tfrac{1}{6}(2\lambda+4\mu-\varepsilon)+J+2]$$
$$\times[\tfrac{1}{6}(4\lambda+2\mu+\varepsilon)-J][2J+1]^{-1}[2J+2]^{-1}\}^{1/2}$$
$$D(\varepsilon J)=\{[\tfrac{1}{6}(2\lambda+4\mu-\varepsilon)-J+1][\tfrac{1}{6}(2\mu-2\lambda+\varepsilon)+J]$$
$$\times[\tfrac{1}{6}(4\lambda+2\mu+\varepsilon)+J+1][2J]^{-1}[2J+1]^{-1}\}^{1/2}.$$

## 3. Multicomponent $q$-coherent states for the quantum algebra $sl_q(3)$

For the $IR(\lambda\mu)$ of $sl_q(3)$, the Hilbert space $V^{(\lambda\mu)}$ spanned by the Elliott-like bases $|(\lambda\mu)\varepsilon JM\rangle$ is composed of $\lambda+\mu+1$ $\varepsilon$-subspaces. Every $\varepsilon$-subspace can be divided into many $J$-subspaces. We now introduce $q$-coherent states $|z\rangle_{\varepsilon J}$ by applying the $q$-exponential operator $E_q(zJ_+)$ on the lowest-weight state $|(\lambda\mu)\varepsilon J-J\rangle$ of every $J$-subspace of the $IR(\lambda\mu)$ for $sl_q(3)$:

$$|z\rangle_{\varepsilon J}=E_q(zJ_+)\,|(\lambda\mu)\varepsilon J-J\rangle$$
$$=\sum_{n=0}^{2J}\left(\frac{[2J]!}{[n]!\,[2J-n]!}\right)^{1/2} z^n|(\lambda\mu)\varepsilon J-J+n\rangle \tag{9}$$

where the $q$-exponential function $E_q(x)$ is defined as

$$E_q(x)=\sum_{n=0}^{\infty}\frac{x^n}{[n]!}. \tag{10}$$

We see from (9) that

$$\langle(\lambda\mu)\varepsilon'J'-J'+n|z\rangle_{\varepsilon J}=\delta_{\varepsilon'\varepsilon}\delta_{J'J}\left(\frac{[2J]!}{[n]!\,[2J-n]!}\right)^{1/2} z^n. \tag{11}$$

Similarly to [13], we introduce the multicomponent $q$-coherent states of $sl_q(3)$: $\{|z\rangle_{\varepsilon J},\ \varepsilon J=\varepsilon_{\min}J_0',\ldots,\varepsilon_{\max}J_0\}$, where $\varepsilon_{\min}=-\lambda-2\mu$, $J_0'=\lambda/2$, $\varepsilon_{\max}=2\lambda+\mu$ and $J_0=\mu/2$ [12]. The scalar product of them is of the form

$$_{\varepsilon'J'}\langle z'|z\rangle_{\varepsilon J}=\delta_{\varepsilon'\varepsilon}\delta_{J'J}B_{2J}(zz'^*) \tag{12}$$

where $B_{2J}(zz'^*)$ is the $q$-binomial

$$B_{2J}(zz'^*)=\{(1+zz'^*)^{2J}\}_q=\sum_{n=0}^{2J}\frac{[2J]!}{[n]!\,[2J-n]!}(zz'^*)^n. \tag{13}$$

One can see from (12) that the $q$-coherent states $|z\rangle_{\varepsilon J}$ in different $\varepsilon$-subspaces or $J$- subspaces are always orthogonal, although those in the same $J$-subspace of $IR(\lambda\mu)$ are not.

Also analogously to [13], the completeness condition of the multicomponent $q$-coherent states of $sl_q(3)$ is defined by

$$\sum_{\varepsilon J}\frac{[2J+1]}{2\pi}\int\frac{d_q^2 z}{B_{2J+2}(|z|^2)}|z\rangle_{\varepsilon J\varepsilon J}\langle z|$$
$$=\sum_{\varepsilon Jn}|(\lambda\mu)\varepsilon J-J+n\rangle\langle(\lambda\mu)\varepsilon J-J+n|=I. \qquad (14)$$

In order to derive (14), we have used the $q$-integration formula which had been proved in [14]:

$$\int_0^\infty \frac{x^n}{B_{2J+2}(x)}d_q x=\frac{[2J-n]!\,[n]!}{[2J+1]!} \qquad 0\leqslant n\leqslant 2J \qquad (15)$$

and

$$\begin{aligned} &d_q^2 z=d_q(|z|^2)\,d\theta \\ &z=r\,e^{i\theta} \qquad 0\leqslant r<\infty \qquad 0\leqslant\theta\leqslant 2\pi \end{aligned} \qquad (16)$$

where the integral over $\theta$ is the usual integration, but the integration over $|z|^2=r^2$ is a $q$-integration.

By virtue of the completeness condition (6), one can obtain the expansion of an arbitrary state in terms of the Elliott-like bases of $sl_q(3)$:

$$|f\rangle=\sum_{\varepsilon Jn}C_n|(\lambda\mu)\varepsilon J-J+n\rangle \qquad (17)$$

where

$$C_n=\langle(\lambda\mu)\varepsilon J-J+n|f\rangle. \qquad (18)$$

On the other hand, by using the completeness condition (14), the expansion of the state $|f\rangle$ in terms of the multicomponent $q$-coherent states of $sl_q(3)$ is

$$|f\rangle=\sum_{\varepsilon J}\frac{[2J+1]}{2\pi}\int\frac{d_q^2 z}{B_{2J+2}(|z|^2)}|z\rangle_{\varepsilon J}f(z^*) \qquad (19)$$

where

$$\begin{aligned} f(z^*)&={}_{\varepsilon J}\langle z|f\rangle \\ &=\sum_{\varepsilon' J' n}C_{n\,\varepsilon J}\langle z|(\lambda\mu)\varepsilon' J'-J'+n\rangle \\ &=\sum_n C_n\left(\frac{[2J]!}{[n]!\,[2J-n]!}\right)^{1/2}(z^*)^n. \end{aligned} \qquad (20)$$

Since $f(z^*)$ may be expanded in a convergent power series, $f(z^*)$ is an entire function of $z^*$.

An expansion analogous to (19) also exists for the adjoint state vectors:

$$\langle g|=\sum_{\varepsilon J}\frac{[2J+1]}{2\pi}\int\frac{d_q^2 z}{B_{2J+2}(|z|^2)}\{g(z^*)\}^*_{\varepsilon J}\langle z| \qquad (21)$$

and

$$\{g(z^*)\}^* = \langle g|z\rangle_{\varepsilon J} \\ = \sum_n C_n^* \left(\frac{[2J]!}{[n]!\,[2J-n]!}\right)^{1/2} z^n . \tag{22}$$

The scalar product of the two states $\langle g|$ and $|f\rangle$ may then be expressed as

$$\langle g|f\rangle = \sum_{\varepsilon J} \left\{\frac{[2J+1]}{2\pi}\right\}^2 \int \frac{B_{2J}(zz'^*)\,\mathrm{d}_q^2 z'\,\mathrm{d}_q^2 z}{B_{2J+2}(|z'|^2)\,B_{2J+2}(|z|^2)} \{g(z'^*)\}^* f(z^*) . \tag{23}$$

On the other hand, using the completeness condition (14) we have

$$\langle g|f\rangle = \sum_{\varepsilon J} \frac{[2J+1]}{2\pi} \int \frac{\mathrm{d}_q^2 z}{B_{2J+2}(|z|^2)} \{g(z^*)\}^* f(z^*) . \tag{24}$$

We may then derive the general identity

$$\frac{[2J+1]}{2\pi} \int \frac{B_{2J}(z^* z')}{B_{2J+2}(|z'|^2)} g(z'^*)\,\mathrm{d}_q^2 z' = g(z^*) . \tag{25}$$

## 4. Expansion of operators in terms of the multicomponent q-coherent states of $sl_q(3)$

A general quantum mechanical operator $T$ may be expressed in terms of its matrix elements connecting the Elliott-like bases of $sl_q(3)$ as

$$T = \sum_{\substack{\varepsilon' J' n \\ \varepsilon J m}} |(\lambda\mu)\varepsilon' J' - J' + n\rangle T_{\varepsilon' J' n, \varepsilon J m} \langle(\lambda\mu)\varepsilon J - J + m| \tag{26}$$

where

$$T_{\varepsilon' J' n, \varepsilon J m} = \langle(\lambda\mu)\varepsilon' J' - J' + n|T|(\lambda\mu)\varepsilon J - J + m\rangle . \tag{27}$$

If we use this expression to calculate the matrix element which connects the two components ${}_{\varepsilon' J'}\langle z'|$ and $|z\rangle_{\varepsilon J}$ of the multicomponent $q$-coherent states of $sl_q(3)$ we find

$${}_{\varepsilon' J'}\langle z'|T|z\rangle_{\varepsilon J} = \sum_{nm} T_{\varepsilon' J' n,\, \varepsilon J m} \left(\frac{[2J']!\,[2J]!}{[n]!\,[m]!\,[2J'-n]!\,[2J-m]!}\right)^{1/2} (z'^*)^n z^m \\ = \mathscr{T}_{\varepsilon' J' \varepsilon J}(z'^*, z) . \tag{28}$$

The magnitudes of the matrix elements $T_{\varepsilon' J' n, \varepsilon J m}$ are dominated by an expression of the form $M[n]^i[m]^k[2J-n]^l[2J-m]^p$ for some fixed positive values of $M$, $i$, $k$, $l$ and $p$. It then follows that the double series (28) converges throughout the finite $z'^*$ and $z$ planes, and the function $\mathscr{T}_{\varepsilon' J' \varepsilon J}(z'^*, z)$ is an entire function of both variables.

To secure the expansion of the operator $T$ in terms of the multicomponent $q$-coherent states of $sl_q(3)$, we may use the representation (14) of the unit operator to write

$$T=\sum_{\substack{\varepsilon'J'\\ \varepsilon J}}\frac{[2J'+1]}{2\pi}\frac{[2J+1]}{2\pi}\int\frac{|z'\rangle_{\varepsilon'J'}\,{}_{\varepsilon'J'}\langle z'|T|z\rangle_{\varepsilon J}\,{}_{\varepsilon J}\langle z|}{B_{2J'+2}(|z'|^2)\,B_{2J+2}(|z|^2)}\,d_q^2z'\,d_q^2z$$

$$=\sum_{\substack{\varepsilon'J'\\ \varepsilon J}}\frac{[2J'+1]}{2\pi}\frac{[2J+1]}{2\pi}\int\frac{|z'\rangle_{\varepsilon'J'}\,\mathcal{T}_{\varepsilon'J'\varepsilon J}(z'^*,z)\,{}_{\varepsilon J}\langle z|}{B_{2J'+2}(|z'|^2)\,B_{2J+2}(|z|^2)}\,d_q^2z'\,d_q^2z. \tag{29}$$

The expansion of operators, as well as of an arbitrary state, in terms of the multicomponent $q$-coherent states of $sl_q(3)$ is a unique one.

For the operator $T^+$, the Hermitian adjoint of $T$, we have

$$(T^+)_{\varepsilon Jm,\varepsilon'J'n}=(T_{\varepsilon'J'n,\varepsilon Jm})^*. \tag{30}$$

If the operator $T$ is Hermitian, the function must satisfy the identity

$$\mathcal{T}_{\varepsilon'J',\varepsilon J}(z'^*,z)=(\mathcal{T}_{\varepsilon J,\varepsilon'J'}(z^*,z'))^*. \tag{31}$$

The law of operator multiplication is easily expressed in terms of the function $\mathcal{T}$. If $T=T_1T_2$ and $\mathcal{T}_1$ and $\mathcal{T}_2$ are the functions appropriate to the latter two operators, we note that

$$\mathcal{T}_{\varepsilon'J',\varepsilon J}(z'^*,z)={}_{\varepsilon'J'}\langle z'|T|z\rangle_{\varepsilon J}$$
$$={}_{\varepsilon'J'}\langle z'|T_1T_2|z\rangle_{\varepsilon J}$$
$$=\sum_{\varepsilon''J''}\frac{[2J''+1]}{2\pi}\int\frac{d_q^2z''}{B_{2J''+2}(|z''^*|^2)}\,{}_{\varepsilon'J'}\langle z'|T_1|z''\rangle_{\varepsilon''J''}\,{}_{\varepsilon''J''}\langle z''|T_2|z\rangle_{\varepsilon J}$$
$$=\sum_{\varepsilon''J''}\frac{[2J''+1]}{2\pi}\int\frac{(\mathcal{T}_1)_{\varepsilon'J',\varepsilon''J''}(z'^*,z'')\,(\mathcal{T}_2)_{\varepsilon''J'',\varepsilon J}(z''^*,z)}{B_{2J''+2}(|z''^*|^2)}\,d_q^2z''. \tag{32}$$

The density operator $\rho$ may be represented by means of a function of two complex variables $R_{\varepsilon'J',\varepsilon J}(z'^*,z)$:

$$\rho=\sum_{\substack{\varepsilon'J'\\ \varepsilon J}}\frac{[2J'+1]}{2\pi}\frac{[2J+1]}{2\pi}\int\frac{|z'\rangle_{\varepsilon'J'}R_{\varepsilon'J',\varepsilon J}(z'^*,z)\,{}_{\varepsilon J}\langle z|}{B_{2J'+2}(|z'|^2)\,B_{2J+2}(|z|^2)}\,d_q^2z'\,d_q^2z \tag{33}$$

where

$$R_{\varepsilon'J',\varepsilon J}(z'^*,z)$$
$$=\sum_{nm}\langle(\lambda\mu)\varepsilon'J'-J'+n|\rho|(\lambda\mu)\varepsilon J-J+m\rangle$$
$$\times\left(\frac{[2J']!\,[2J]!}{[n]!\,[m]!\,[2J'-n]!\,[2J-m]!}\right)^{1/2}(z'^*)^n z^m. \tag{34}$$

The statistical average of an operator $T$ is given by the trace of the product $\rho T$,

$$\mathrm{tr}(\rho T)=\sum_{\varepsilon J}\frac{[2J+1]}{2\pi}\int\frac{\mathrm{d}_q^2 z}{B_{2J+2}(|z|^2)}\,{}_{\varepsilon J}\langle z|\rho T|z\rangle_{\varepsilon J}$$

$$=\sum_{\substack{\varepsilon' J'\\ \varepsilon J}}\frac{[2J'+1]}{2\pi}\frac{[2J+1]}{2\pi}\int\frac{R_{\varepsilon J,\varepsilon' J'}(z^*,z')\,\mathscr{T}_{\varepsilon' J',\varepsilon J}(z'^*,z)}{B_{2J+2}(|z|^2)\,B_{2J'+2}(|z'|^2)}\,\mathrm{d}_q^2 z'\,\mathrm{d}_q^2 z\,. \tag{35}$$

If $T$ is a unit operator $T=1$, we have

$$\mathscr{T}_{\varepsilon' J',\varepsilon J}(z'^*,z)={}_{\varepsilon' J'}\langle z'|T|z\rangle_{\varepsilon J}$$
$$=\delta_{\varepsilon\varepsilon'}\,\delta_{JJ'}B_{2J}(zz'^*)\,. \tag{36}$$

The trace of $\rho$ itself, which must be normalized to unity, is

$$\mathrm{tr}\,\rho=1$$
$$=\sum_{\varepsilon J}\left(\frac{[2J+1]}{2\pi}\right)^2\int\frac{B_{2J}(zz'^*)\,\mathrm{d}_q^2 z'\,\mathrm{d}_q^2 z}{B_{2J+2}(|z|^2)\,B_{2J+2}(|z'|^2)}\,R_{\varepsilon J,\varepsilon J}(z^*,z')\,. \tag{37}$$

Since $R_{\varepsilon J,\varepsilon J}(z^*,z')$ is an entire function of $z^*$, we may use (25) to carry out the integration over the $z$-plane. In this way we see that the normalization condition on $R$ is

$$\sum_{\varepsilon J}\frac{[2J+1]}{2\pi}\int\frac{R_{\varepsilon J,\varepsilon J}(z^*,z)}{B_{2J+2}(|z|^2)}\,\mathrm{d}_q^2 z=1\,. \tag{38}$$

## 5. Inhomogeneous differential realization

It has been proved that the quasi-exactly solvable problems of quantum mechanics are related to the inhomogeneous differential realization of Lie (super) algebra [15, 16]. So we will study this realization of the quantum algebra $sl_q(3)$ here.

The action of the generators of $sl_q(3)$ on its multicomponent $q$-coherent states is

$$X|z\rangle_{\varepsilon J}=\sum_{\varepsilon' J'}(R(X))_{\varepsilon J,\varepsilon' J'}|z\rangle_{\varepsilon' J'} \tag{39}$$

where $X=Q, J_0, J_\pm, T_{\pm 1/2}$ and $V_{\pm 1/2}$. $R(X)$ is the matrix representation of the generator $X$ in the multicomponent $q$-coherent state space. Its elements are $(R(X))_{\varepsilon J,\varepsilon' J'}$; here the row index $\varepsilon J$, as well as the column index $\varepsilon' J'$, is an ordered pair.

Then we can obtain that

$$(R(Q))_{\varepsilon J,\varepsilon' J'}=\delta_{\varepsilon\varepsilon'}\delta_{JJ'}\,\varepsilon$$

$$(R(J_0))_{\varepsilon J,\varepsilon' J'}=\delta_{\varepsilon\varepsilon'}\delta_{JJ'}\left(-J+z\frac{\mathrm{d}}{\mathrm{d}z}\right)$$

$$(R(J_+))_{\varepsilon J,\varepsilon' J'}=\delta_{\varepsilon\varepsilon'}\delta_{JJ'}\frac{\mathrm{d}}{\mathrm{d}_q z}$$

$$(R(J_-))_{\varepsilon J,\varepsilon' J'}=\delta_{\varepsilon\varepsilon'}\delta_{JJ'}\left(-z^{2J+2}\frac{\mathrm{d}}{\mathrm{d}_q z}z^{-2J}\right)$$

$$
\begin{aligned}
&(R(T_{1/2}))_{\varepsilon J,\varepsilon' J'} \\
&\quad =\delta_{\varepsilon'\varepsilon+3}\delta_{J'J+1/2}A(\varepsilon J)\{[2J+1]\}^{-1/2}\frac{\mathrm{d}}{\mathrm{d}_q z} \\
&\quad\quad -\delta_{\varepsilon'\varepsilon+3}\delta_{J'J-1/2}B(\varepsilon J)\{[2J]\}^{1/2} \\
&(R(T_{-1/2}))_{\varepsilon J,\varepsilon' J'} \\
&\quad =-\delta_{\varepsilon'\varepsilon+3}\delta_{J'J+1/2}A(\varepsilon J)\{[2J+1]\}^{-1/2}z^{2J+2}\frac{\mathrm{d}}{\mathrm{d}_q z}z^{-2J-1} \\
&\quad\quad +\delta_{\varepsilon'\varepsilon+3}\delta_{J'J-1/2}B(\varepsilon J)\{[2J]\}^{1/2}z \\
&(R(V_{1/2}))_{\varepsilon J,\varepsilon' J'} \\
&\quad =\delta_{\varepsilon'\varepsilon-3}\delta_{J'J+1/2}C(\varepsilon J)\{[2J+1]\}^{-1/2}\frac{\mathrm{d}}{\mathrm{d}_q z} \\
&\quad\quad +\delta_{\varepsilon'\varepsilon-3}\delta_{J'J-1/2}D(\varepsilon J)\{[2J]\}^{1/2} \\
&(R(V_{-1/2}))_{\varepsilon J,\varepsilon' J'} \\
&\quad =-\delta_{\varepsilon'\varepsilon-3}\delta_{J'J+1/2}C(\varepsilon J)\{[2J+1]\}^{-1/2}z^{2J+2}\frac{\mathrm{d}}{\mathrm{d}_q z}z^{-2J-1} \\
&\quad\quad -\delta_{\varepsilon'\varepsilon-3}\delta_{J'J-1/2}D(\varepsilon J)\{[2J]\}^{1/2}z
\end{aligned}
\tag{40}
$$

where $\mathrm{d}/\mathrm{d}z$ is the usual differential operator, and $\mathrm{d}/\mathrm{d}_q z$ is the $q$-differential operator with respect to the complex variable $z$. The $q$-derivative is defined to be [17, 18]

$$
\frac{\mathrm{d}}{\mathrm{d}_q z}f(z)=\frac{f(q^{-1}z)-f(qz)}{q^{-1}z-qz}.
\tag{41}
$$

We have the following $q$-derivative formulae:

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}_q z}\{f(z)\pm g(z)\}=\frac{\mathrm{d}}{\mathrm{d}_q z}f(z)\pm\frac{\mathrm{d}}{\mathrm{d}_q z}g(z) \\
&\frac{\mathrm{d}}{\mathrm{d}_q z}\{f(z)g(z)\}=\left\{\frac{\mathrm{d}}{\mathrm{d}_q z}f(z)\right\}g(q^{-1}z)+f(qz)\frac{\mathrm{d}}{\mathrm{d}_q z}g(z) \\
&\frac{\mathrm{d}}{\mathrm{d}_q z}\left\{\frac{f(z)}{g(z)}\right\}=\{g(q^{-1}z)g(qz)\}^{-1} \\
&\qquad\qquad\times\left\{g(q^{-1}z)\frac{\mathrm{d}}{\mathrm{d}_q z}f(z)-f(qz)\frac{\mathrm{d}}{\mathrm{d}_q z}g(z)\right\}.
\end{aligned}
\tag{42}
$$

It may be proved that the matrix operators $R(X)$, whose elements are shown by (41), give rise to an inhomogeneous differential realization of $sl_q(3)$.

## 6. Concluding remarks

We have constructed the multicomponent $q$-coherent states and obtained the inhomogeneous differential realization for the quantum algebra $sl_q(3)$. It is obvious that this method may be generalized to arbitrary Lie algebra, Lie superalgebra and quantum algebra which contains the subalgebra $su(2)$. A straightforward example is the Lie algebra $su(3)$, which is the classical counterpart for $sl_q(3)$. The vector coherent states of $su(3)$ have been given by Hecht [19]. It is also possible to construct the multicomponent coherent states of $su(3)$ by means of the method represented in this paper.

## Acknowledgment

This work was supported in part by the Fund of the Science and Technology Committee of Hunan province.

## References

[1] Klauder J R and Skagerstam B S (ed) 1985 *Coherent States* (Singapore: World Scientific)
[2] Schrödinger E 1926 *Naturwissenschaften* **14** 664
[3] Glauber R J 1963 *Phys. Rev.* **130** 2529; 1963 *Phys. Rev.* **131** 2766
[4] Perelomov A M 1972 *Commun. Math. Phys.* **26** 222; 1975 *Commun. Math. Phys.* **44** 197
[5] Gilmore R 1972 *Ann. Phys.* **74** 391
Gilmore R, Bowden C M and Narducci L M 1975 *Phys. Rev.* A **12** 1019
[6] Zhang Wei-Min, Feng Da Hsuan and Gilmore R 1990 *Rev. Mod. Phys.* **62** 867
[7] Biedenharn L C 1989 *J. Phys. A: Math. Gen.* **22** L873
[8] Macfarlane A J 1989 *J. Phys. A: Math. Gen.* **22** 4581
[9] Quesne C 1991 *Phys. Lett.* **153A** 303
[10] Jimbo M 1985 *Lett. Math. Phys.* **10** 63; 1986 *Lett. Math. Phys.* **11** 247
[11] Yu Zurong 1991 *J. Phys. A: Math. Gen.* **24** L399
[12] Elliott J P 1958 *Proc. R. Soc. London* A **245** 128; *Proc. R. Soc. London* A **245** 562
[13] Kuang Le-Men and Zeng Gao-Jian 1993 *J. Phys. A: Math. Gen.* **26** 913
[14] Kuang Le-Man 1992 *J. Phys. A: Math. Gen.* **25** 4827
[15] Turbiner A V 1988 *Commun. Math. Phys.* **118** 467
[16] Shifman M A and Turbiner A V 1989 *Commun. Math. Phys.* **126** 347
[17] Gray R W and Nelson C A 1990 *J. Phys. A: Math. Gen.* **23** L945
[18] Bracken A J, McAnally D S, Zhang R B and Gould M D 1991 *J. Phys. A: Math. Gen.* **24** 1379
[19] Hecht K T 1987 *Nucl. Phys.* A **475** 276